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# THE EFFECT OF TENSION STIFFENING ON THE BEHAVIOUR OF R/C BEAMS

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## ABSTRACT

Tension stiffening, is attributed to the fact that concrete does not crack suddenly and completely but undergoes progressive microcracking(strain softening). Immediately after first cracking, the intact concrete between adjucent primary cracks carries considerable tensile force due to the bond between the steel and the concrete. The bending stiffness of the member is considerably greater than that based on a fully cracked section, where concrete in tension is assumed to carry zero stress. This tension stiffening effect may be significant in the service load performance of beams and slabs. This phenomenon is also effective in long-term behaviour of concrete members due to the creep and shrinkage of the concrete. Various methods have been proposed to account for tension stiffening in the analysis of concrete structures. An approach for modelling tension stiffening is to assume that an area of concrete located at the tensile steel level is effective in providing stiffening. In this paper a simple formulation is proposed for study of short-term and long-term behaviour of reinforced concrete beams and one-way slabs considering tension stiffening effect. The proposed formula is validated with experimental results and some numerical examples are worked out.

**Keywords**: Concrete beams; tension stiffening; long-term behaviour

## 1. INTRODUCTION

The bending stiffness of reinforced concrete beams under service loads is considerably smaller than the stiffness calculated on the basis of uncracked cross sections. This is beacause the beam contains numerous tensile cracks. Yet, at the same time, the stiffness is significantly higher than that calculated when the tensile resistance of concrete is neglected. This phenomenon, often termed tension stiffening, is attributed to the fact that concrete does not crack suddenly and completely but undergoes progressive microcracking [1]. Immediately after first cracking, the intact concrete between adjucent primary cracks carries considerable tensile force, mainly in the direction of the reinforcement, due to the bond between the steel and the concrete. The average tensile stress in the concrete is a significant

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percentage of the tensile strength of concrete. The steel stress is a maximum at a crack, where the steel carries the entire tensile force, and drops to a minimum between cracks. The bending stiffness of the member is considerably greater than that based on a fully cracked section, where concrete in tension is assumed to carry zero stress. This tension stiffening effect may be significant in the service load performance of beams and slabs [2].

## 2. MODELS FOR TENSION STIFFENING

Various methods have been proposed to account for tension stiffening in the analysis of concrete structures. These range from simple empirical estimates of the flexural rigidity of a member [3-5] to assumed unloading stress-strain relationship for concrete in tension [6-8]. Techniques involving an adjustment to the stiffness of the tensile steel to account for tension stiffening have also been used [6,9]. An alternative approach for modelling tension stiffening is to assume that an area of concrete located at the tensile steel level is effective in providing stiffening [1,4,10,11]. Figure 1 shows an average cross-section of a singly reinforced member. The properties of this average section are between those of the fully-cracked cross section and the uncracked cross section between the primary cracks. The tensile concrete area $A_{ct}$, which is assumed to contribute to the beam stiffness after cracking, depends on the magnitude of the maximum applied moment M, the area of the tensile reinforcement $A_{st}$, the amount of concrete below the neutral axis, the tensile strength of the concrete (the cracking momemt $M_c$), and the duration of sustained load.

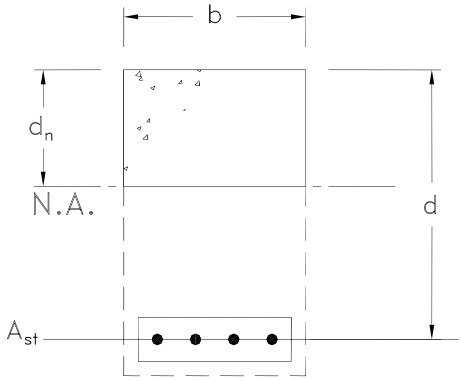


Figure 1. Average section after cracking

Bazant and Byung proposed a simplified equivalent transformed cross section, taken from their proposed model which is drived from the intrinsic material properties of concrete, particularely the strain-softening properties [1]. In this simplified model, which is used in this study, the tensile resistance of concrete distributed over the tension side of the neutral axis is neglected and an equivalent tensile area $A_{eq}$ and an equivalent tensile stress of concrete in this area $\sigma_{eq}$, which would yield about the same beam curvature $\kappa$, isdetermined. The centroid of this equivalent area coincides with that of tensile reinforcement, Figure 2.

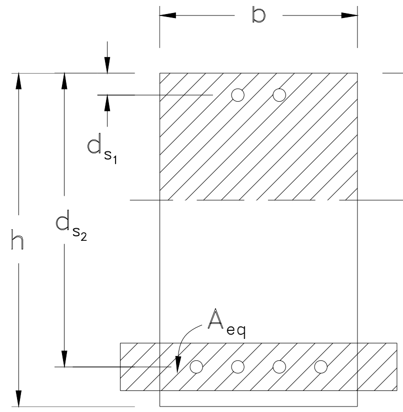


Figure 2. Equivalent transformed cross section

The equivalent tensile area can be obtained from:

$$A_{eq} = [b(d_n)^2/2 + nA_{s1}(d_n - d_{s1})] \times (d_{s2} - d_n)^{-1} - nA_{s2} \tag{1}$$

In which $A_{s1}$ and $A_{s2}$ are the area of compression and tension reinforcement, respectively, $n = E_s / E_c$ and difinition of the rest of parameters are shown in Figure 2. The value of $d_n$ needed in Eqation 1 has to be calculated from the condition of the same carvature $\kappa$. Two cases is distinguished depending on whether the tensile stress in concrete at tensile face is zero or finite[1]. With considering these cases, $A_{eq}$ and $d_n$ could be evaluated in an iterative manner for a given bending moment. Once $d_n$ and $A_{eq}$ are determined, the inertia moment of the transformed cross section can be evaluated.

## 3. SHORT-TERM ANALYSIS OF CROSS SECTIONS

Consider the equivalent transformed croos section in Figure 2. The top surface of the cross section is selected as the reference surface. The position of the centroidal axis depends on the quantity of bonded reinforcement and varies with time owing to the gradual development of creep and shrinkage in the concrete. Therefore, it is convenient to select a fixed reference point that can be used in all stages of analysis [2].

### *3.1 Uncracked Section*

In Figure 3, the strain at a depth $y$ below the top of the cross section is defined in terms of the top fibre strain $\varepsilon_{0i}$ and the initial curvature $\kappa_i$, as follows:

$$\varepsilon_i = \varepsilon_{0i} + y\,\kappa_i \tag{2}$$

the initial concrete stress at $y$ below the top fibre is:

$$\sigma_i = E_c\,\varepsilon_i = E_c\,(\varepsilon_{0i} + y\ \kappa_i) \tag{3}$$

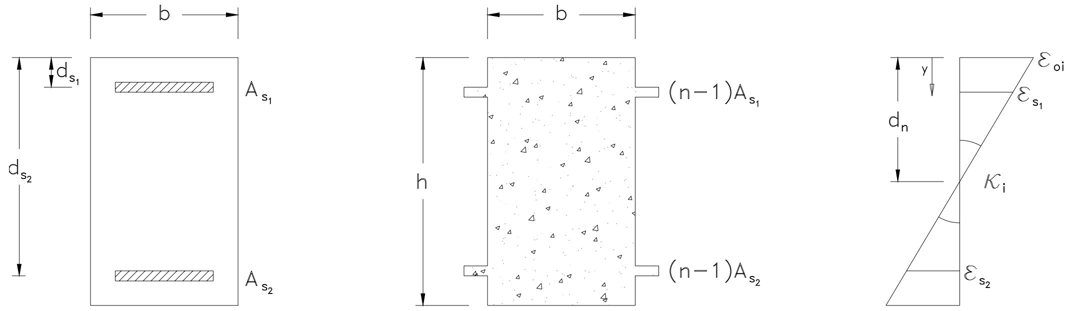


Figure 3. Uncracked section analysis

Integrating the stress block over the depth of the section, horizontal equilibrium requires that:

$$\begin{aligned} N_i &= \int \sigma_i \,\mathrm{d}A \\ &= E_c\,\varepsilon_{0i} \int \mathrm{d}A + E_c\,\kappa_i \int y\,\mathrm{d}A \\ &= E_c\,\varepsilon_{0i}\,A + E_c\,\kappa_i\,B \end{aligned} \tag{4}$$

where $A$ (= ∫ d$A$) is the area of the transformed section and $B$ (= ∫ $y$d$A$) is the first moment of the stress block about the top surface of the section. If the first moment of the stress block about the top fibre is integrated over the depth of the section, the resultant moment about the top surface, $M_i$ , is found. Therefore,

$$\begin{aligned} M_i &= \int \sigma_i\ y\,\mathrm{d}A \\ &= E_c\,\varepsilon_{0i} \int y\,\mathrm{d}A + E_c\,\kappa_i \int y^2\,\mathrm{d}A \\ &= E_c\,\varepsilon_{0i}\,B + E_c\,\kappa_i\,\bar{I} \end{aligned} \tag{5}$$

where $\bar{I}$ ( = ∫ $y^2$ d$A$ ) is the second moment of the transformed area about the top surface of the transformed section. By rearranging Eqs. 4 and 5, expressions are obtained for the initial top fibre strain and curvature :

$$\varepsilon_{0i} = B\,M_i \,/\, [E_c\,(B^2 - A\,\bar{I}\,)] \tag{6}$$

$$\kappa_i = -A\,M_i \,/\, [E_c\,(B^2 - A\,\bar{I}\,)] \tag{7}$$

*3.2 Cracked Section*

The instantaneous strains and stresses on a cracked section are shown in Figure 4. Horizontal equilibrium dictates that:

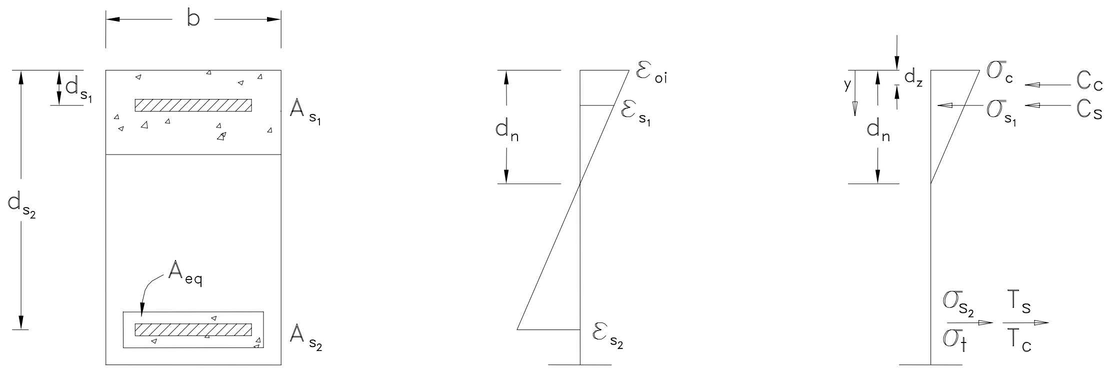


Figure 4. Cracked section analysis

$$T_s - C_c - C_s + T_c = 0 \tag{8}$$

and moment equilibrium requires that

$$M = C_c\, d_z + C_s\, d_{s1} - T_c\, d_{s2} - T_s\, d_{s2} \tag{9}$$

where $C_c$ , $C_s$ , $T_c$ , and $T_s$ can be expressed as functions of $d_n$ and $\varepsilon_{0i}$ :

$$C_c = \sigma_{0i}\, bd_n /2 = E_c\, \varepsilon_{0i}\, bd_n /2 \tag{10}$$

$$C_s = E_s\, A_{s1}\, [\varepsilon_{0i}\, (d_n - d_{s1})/d_n] \tag{11}$$

$$T_s = E_s\, A_{s2}\, [\varepsilon_{0i}\, (d_{s2} - d_n)/d_n] \tag{12}$$

$$T_c = \sigma_{eq}\, A_{eq} = E_c\, A_{eq}\, [\varepsilon_{0i}\, (d_{s2} - d_n)/d_n] \tag{13}$$

By substituting Eqs. (9-13) into Eqs. (8) and (9) and solving the simultaneous equations, $\varepsilon_{0i}$ and $d_n$ are found with an iterative manner. Based on the values of $\varepsilon_{0i}$ and $d_n$ the curvature can be calculated:

$$\kappa_i = - \varepsilon_{0i} / d_n \tag{14}$$

## 4. TIME-DEPENDENT ANALYSIS OF CROSS-SECTIONS

During any time period, creep and shrinkage strains develop in the concrete. The time-dependent change of strain at any depth y below the top of the cross section, $\Delta\varepsilon$, may be expressed in terms of the change in top fibre strain, $\Delta\varepsilon_0$ , and the change of curvature, $\Delta\kappa$ :

$$\Delta\varepsilon = \Delta\varepsilon_0 + y\ \Delta\kappa \tag{15}$$

The increments of top fibre strain, $\Delta\varepsilon_0$ , and curvature, $\Delta\kappa$ , may be obtained from the following equations [2]:

$$\Delta\varepsilon_0 = (\overline{B}_e\,\delta M - \bar{I}_e\,\delta N) / [\bar{E}_e\,(\overline{B}_e^{\,2} - \bar{A}_e\,\bar{I}_e)] \tag{16}$$

$$\Delta\kappa = (\overline{B}_e\,\delta N - \bar{A}_e\,\delta M) / [\bar{E}_e\,(\overline{B}_e^{\,2} - \bar{A}_e\,\bar{I}_e)] \tag{17}$$

where $\bar{A}_e$ is the area of the age-adjusted equivalent transformed section and $\overline{B}_e$ and $\bar{I}_e$ are the first and second moments of the area of the age-adjusted equivalent transformed section about the top surface. For the determination of $\bar{A}_e$ , $\overline{B}_e$ , and $\bar{I}_e$ the age-adjusted effective modulus $\bar{E}_e$ is used [12]. Therefore, the total strain and curvature may be obtained from:

$$\varepsilon = \varepsilon_{0i} + \Delta\varepsilon \tag{18}$$

$$\kappa = \kappa_i + \Delta\kappa \tag{19}$$

The deflection δ at any point along a beam can be calculated by integrating the curvature $\kappa(x)$ over the length of the beam:

$$\delta = \iint \kappa(x)\,\mathrm{d}x\,\mathrm{d}x \tag{20}$$

Consider the beam shown in Figure 5. If the variation in curvature along the member, subjected to uniform load $q$ is parabolic, then the deflection at mid-span, $\delta_C$ , is given by [2]:

$$\delta_c = (\kappa_A + 10\,\kappa_C + \kappa_B)\,L^2/96 \tag{21}$$

where $\kappa_A$ and $\kappa_B$ are the curvature at each end of the span and $\kappa_C$ is the curvature at mid-span.

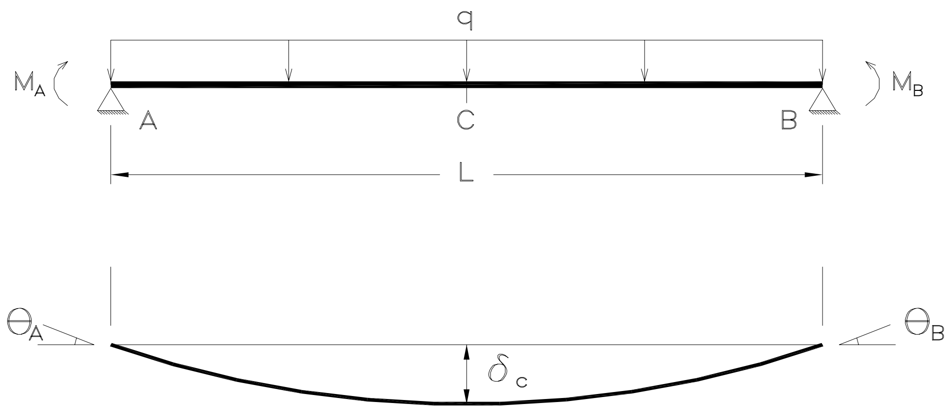

Figure 5. Deflection of a typical beam

The deflections were computed as just detailed for two simply supported test beams reported by Bakoos et al. [13] and Hollington [14]. Dimension of beam 1B2, tested by Bakoos et al. was L×B×H = 3750×100×150mm, and was reinforced with two 12 mm deformed bars at an effective depth of 130 mm. The beam 1B2 was subjected to sustained load consisting of two point loads each of 2.6 kN applied at the third points of the span at 28 days after casting. 28-day compressive strength of concrete was 39.0 MPa. Full details of the test is reported in refrence 14. Midspan deflection, obtained from test and calculated from proposed model are shown in Figure 6. It can be seen that the agreement between theory and experiment is good. The results of proposed model is also plotted in Figure 7 along with deformations obtained experimentally by Hollington [14]. As seen, the results are in good agreement.

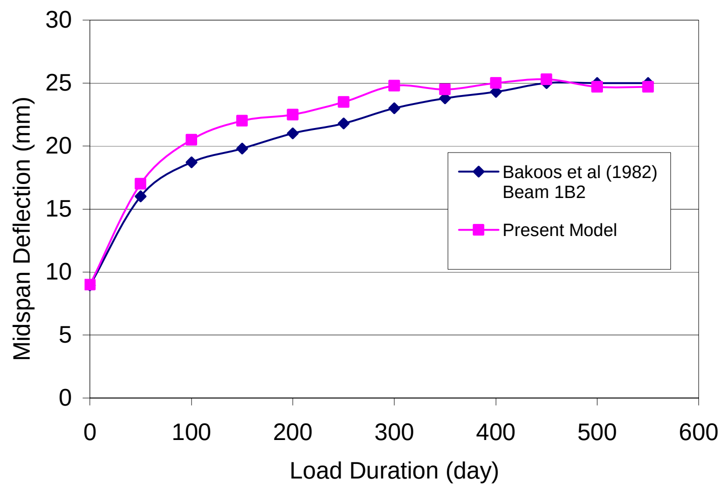


Figure 6. Comparison of Deflection –Time Curves

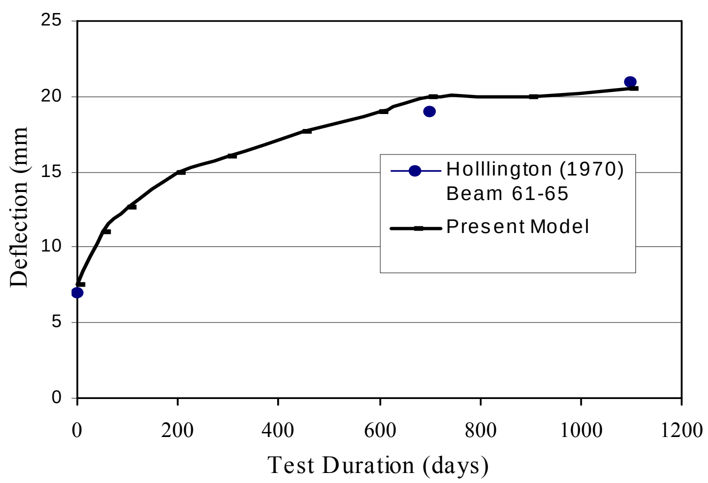

Figure 7. Comparison of time-dependent deflection

## 5. WORKED EXAMPLE

*5.1 Cross-sectional analysis*

The above simple formulation may be used to study the short-term and long-term behaviour of reinforced concrete beams and one-way slabs. The proposed formulation here is used to study the cross-sectional behavior of a one-way reinforced concrete slab, shown in Figure 8, with considering the tension stiffening effect (TS) and without this effect (NTS).

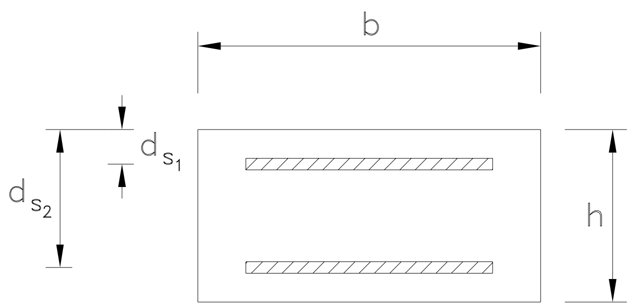


Figure 8. Cross-section of one-way reinforced concrete slab

Considered Cross-sectional properties are: $b$ = 1000 mm, $h$ = 180 mm, $As_1$ = 452 mm$^2$, $As_2$= 1017 mm$^2$ , $ds_1$ = 30 mm, $ds_2$ = 150 mm, $f'_c$ = 21 MPa, $f_y$ =300 MPa, creep coefficient $\varphi$ = 3, shrinkage strain $\varepsilon_{sh}$ = -450×10$^{-6}$. In Figure 9, the instantaneous curvature is plotted in two cases TS and NTS. As seen, with increase in magnitude of applied bending moment, the effect of tension stiffening decreases, however, for moments close to cracking moment of cross-section (service-load range) the instantaneous curvature could be about 22 percent overestimated without considering the TS effect. The same conclusion can be obtained in case of total curvature,i.e., the sum of instantaneous and long-term curvature, Figure 10.

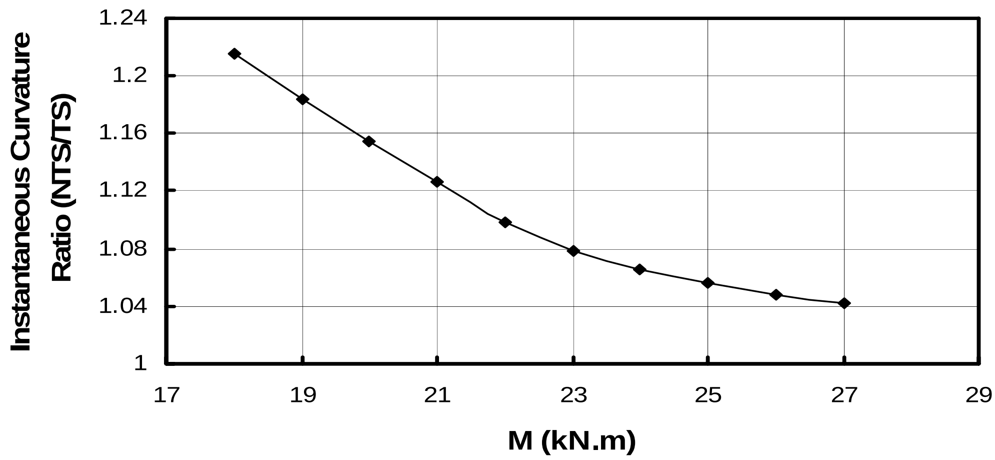

Figure 9. Comparison of instantaneous curvature

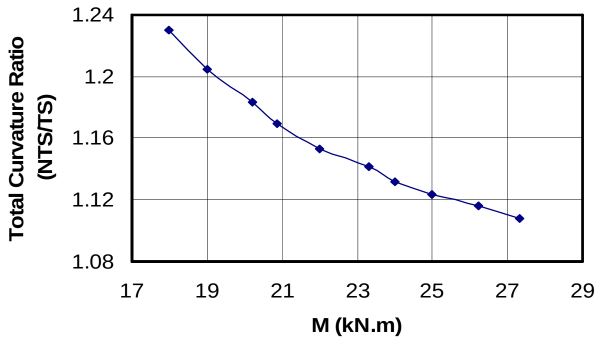


Figure 10. Comparison of total curvature

The tensile reinforcement percentage, P = $A_s$/bd, in above cross-section is equal to 0.68. In Figures 11 and 12, the above cross-section is considered with lower P (=0.5%) and higher P (=1%), respectively. As shown, in lightly reinforced section the instantaneous curvature could be about 25% overestimated without considering tension stiffening effect and in more heavily reinforced section (P=1%) curvature overestimation is about 18%. These figures show that benefits of considering tension stiffening remain for practical levels of reinforcement.

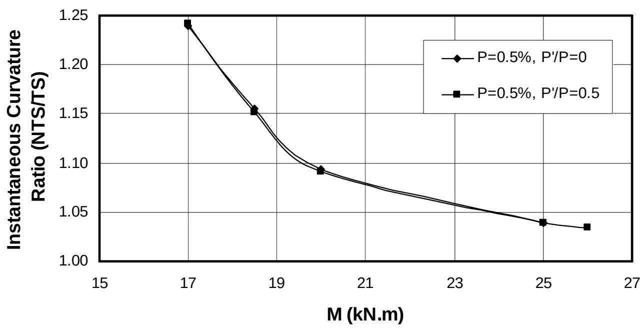


Figure 11. Comparison the effect of reinfocement percentage (P=0.5%)

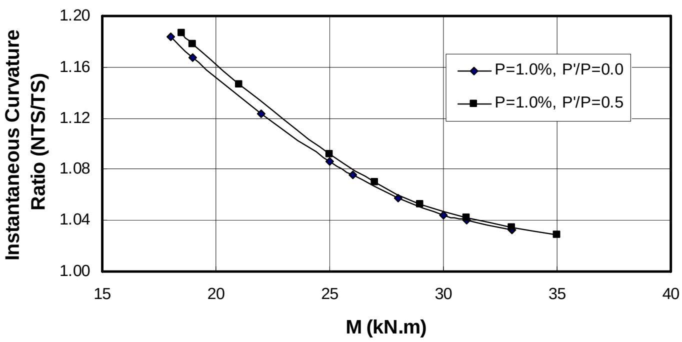

Figure 12. Comparison the effect of reinforcement percentage (P=1%)

Curves in Figures 11 and 12 are plotted for different values of compression reinforcement percentage , i.e., P' = 0.0 and 0.5%. It can be seen that tension stiffening effect depends mainly on the percentage of tensile reinforcement and on the bending moment value but is almost independent of the percentage of compression reinforcement.

After short-term and long-term analysis of cross-section, the calculated concrete compressive stress is shown in Figure 13. As seen, the concrete compressive stress is increased with increse in applied bending moment, however, without considering TS effect, the concrete compressive stress may be overestimated by 20 percent under service loads. With increase in bending moment, concrete undergoes progressive microcracking and therefore the equivalent tensile concrete area, $A_{eq}$, reduces and the long-term compressive stress in concrete increases, Figure 12.

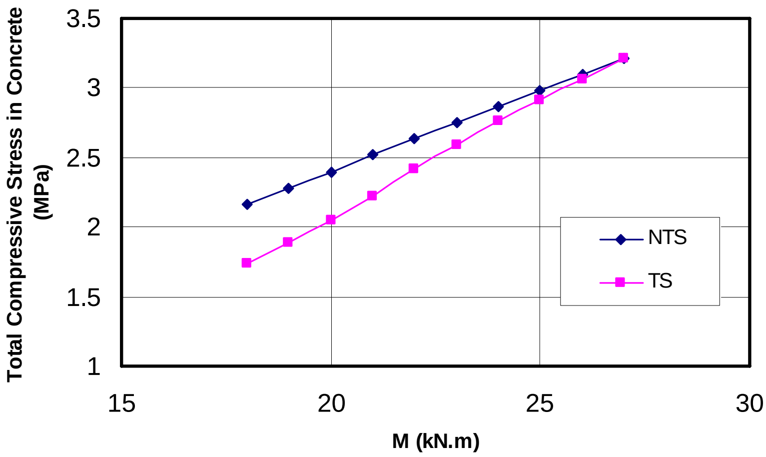


Figure 13. Comparison of concrete compressive stress

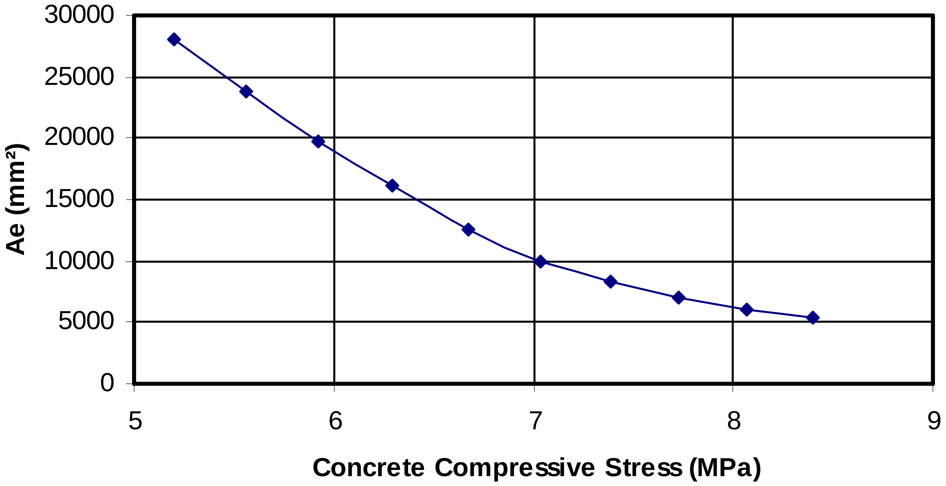

Figure 14. The variation of concrete compressive stress with $A_{eq}$

The variation of $A_{eq}$ with bending moment and the percentage of tensile and compression reinforcement is shown in Figures 15 and 16. These plots show that the effect of the ratio of the cross-section areas of compression and tensile reinforcement, P'/P, on $A_{eq}$ is negligible but equivalent area significantly depends on the percentage of tensile reinforcement and on the bending moment value relative to the cracking moment.

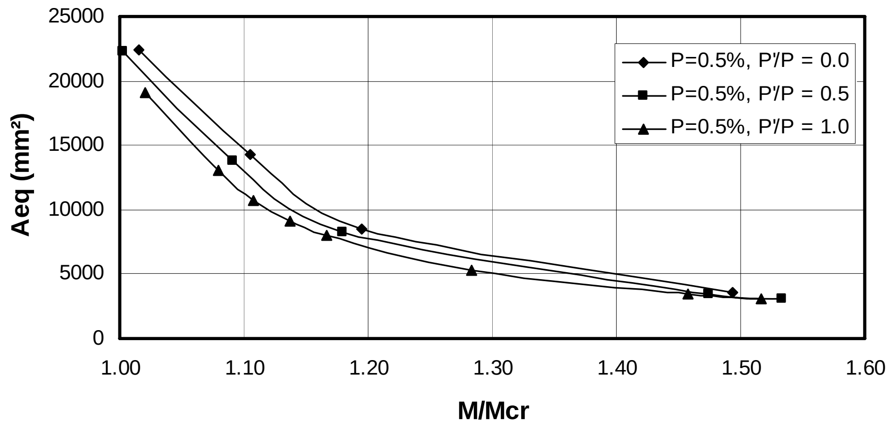


Figure 15. Comparison of the effect of reinforcement percentage on $A_{eq}$ (P=0.5%)

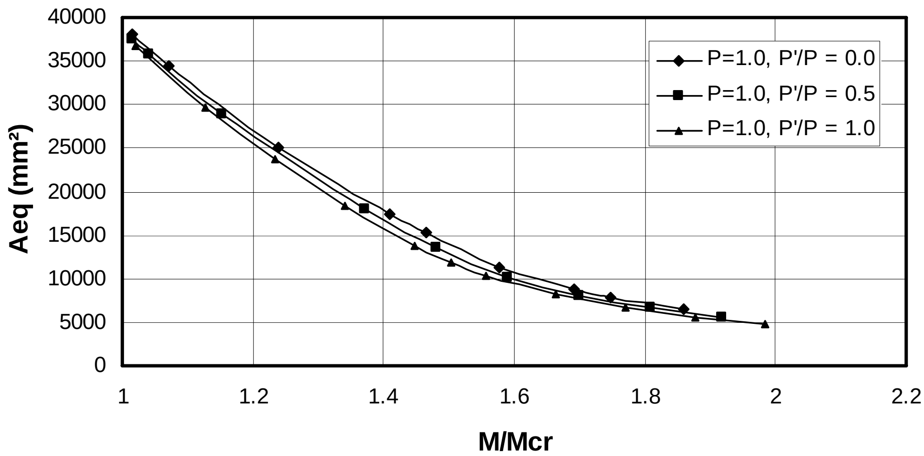


Figure 16. Comparison of the effect of reinforcement percentage on $A_{eq}$ (P=1%)

*5.2 Continuous one-way slab*

In Table 1 the midspan displacements of a four-span one-way slab , subjected to uniformly distributed load q is given, Figure 17. Short-term ($\delta_i$) and long-term displacements(total displacements $\delta_T$ ) are calculated in two cases, TS and NTS. The differences in percent is tabulated. Slab properties are:

$b$ = 1000 mm, $h$ = 180 mm, $d$ =150 mm. At support B and support D: $A_{s1}$ = 452 mm$^2$, $A_{s2}$= 905 mm$^2$. At support C: $A_{s1}$ = 452 mm$^2$, $A_{s2}$ = 1017 mm$^2$. At midspans: $A_{s1}$ = 452 mm$^2$, $A_{s2}$ = 678 mm$^2$. $f'_c$ = 21 MPa , $f_y$ = 300 MPa, $E_s$ = 2x 10$^5$ MPa, $q$ = 8.82 N/mm.

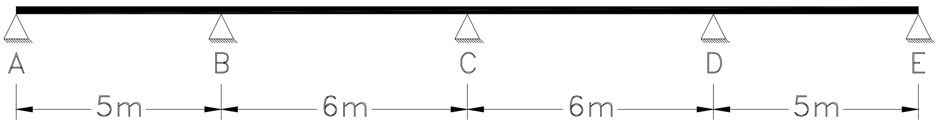


Figure 17. Four-span one-way slab

Table 1. Midspan displacements

| Midspan displacement (mm) | | | | | | |
|---|---|---|---|---|---|---|
| | Span AB | | | Span BC | | |
| | **TS** | **NTS** | **Diff. (%)** | **TS** | **NTS** | **Diff. (%)** |
| $\delta_i$ | 16.8 | 19.7 | 14.7 | 28.3 | 32.3 | 12.4 |
| $\delta_T$ | 28.8 | 36.7 | 21.5 | 47.9 | 59 | 18.8 |

As seen in Table 1, the midspan instantaneous deflection of spans AB and BC, if tension stiffening effect considered, is 14.7 and 12.4 percent smaller, respectively. After long-term analysis, the total midspan deflection of spans AB and BC is 21.5 and 18.8 percent smaller, respectively. This means that ignoring the tension stiffening effect could lead to a considerable overestimation of displacements.

## 6. SUMMARY AND CONCLUSIONS

It is well known that after cracking the concrete between the cracks carries tension and hence stiffens the response of a reinforced concrete member subjected to tension. This stiffening effect, after cracking, is refered to as tension stiffening. A simple formulation is proposed for study of short-term and long-term behaviour of reinforced concrete beams and one-way slabs considering tension stiffening effect. Based on the obtained results the theoretical predictions agree well with experimental results.

The results of worked examples on a one-way slab shows that for moments close to cracking moment of cross-section (service-load range), the curvature, concrete compressive stress, and midspan displacement may be overestimated by 20 percent without considering the tension stiffening effect. Therefor, considering the effect of tension stiffening in short-term and long-term study of concrete beams and one-way slabs could lead to a more reasonable assessment of their behaviour.

## REFERENCES

1. Zant ZP, Byung HO. Deformation of progressively cracking reinforced concrete beams. *ACI Journal*, No. 3, **81**(1984) 268-78.
2. Gilbert RI. Time Effects in Concrete Structures, Elsevier,London, 1988.
3. Branson, D.E., Design procedures for computing deflection. *ACI Journal*, Proc. No. 9, **65**(1968) 730-42.
4. Gilbert RI. Deflection calculations for concrete beams. *Civil Engineering Transactions, Institution of Engineers, Australia,* No. 2, **CE25**(1983) 128-34.
5. Comite Euro-International du Beton (CEB). Fissuration et Deformation, Manual de CEB, Ecole Polytechnique Federale de Lausanne, Suisse, 1983.
6. Gilbert RI, Warner RF. Tension stiffening in reinforced concrete slab. *Journal of the Structural Division, ASCE*, No. 12, **104**(1978) 1885-900.
7. Lin CS, Scordelis AC. Nonlinear analysis of R.V. shells of general form. *Journal of the Structural Division, ASCE*, No. 3, **101**(1975) 523-38.
8. Scanlon A, Murray DW. Time dependent reinforced concrete slab deflections. *Journal of the Structural Division, ASCE*, No. 9, **100**(1974) 1911-24.
9. Bridge RQ, Smith RG. Tension stiffening model for reinforced concrete members, *8th Australian Conference on the Mechanics of Structures and Materials, University of Newcastle,* 1982, pp. 41-6.
10. Clark LA, Speirs DM. Tension Stiffening in Reinforced Concrete Beam and Slabs Under Short-term Loads, *Technical Report 42.521, Cement and Concrete Association,* London, 1978.
11. Rao PS, Subrahmanyam BV. Trisegmental moment-curvature relations for reinforced concrete members. *ACI Journal,* No. 5, **70**(1973) 346-51.
12. Bazant ZP. Mathematical models for creep and shrinkage in concrete structures. *Creep and Shrinkage in Concrete Structure, John Wiley & Sons, Chichester*, 1982.
13. Bakoss SL, Gilbert RI, Faulkes KA, and Pulmano VA. Long-term tests on reinforced concrete beams, *UNICIV Report No R-204, The University of NSW,* Sydney, 1982.
14. Hollington MR. A series of long-term tests to investigate the deflection of a representative precast concrete floor component. *Technical Report No. TRA 442, Cement and Concrete Association,* London, 1970.